Pacman Product for Polysigned numbers

Precedent

The author of the polysigned numbers, ゴールデン・P・ティム, describes other anatomy of numbers, different than the usual one where the word sign is related to the trichotomy of real numbers and the sign function. In Golden's system, one has a essential pillar, the term. The term is composed by two parts of distinct nature, the first, called sign, indicates direction, and the second is called magnitude. Both parts do not mix. In -125, the section - indicates direction, and the section 125 indicates magnitude.

In addition, Golden adds that the directions and axes in his system are not orthogonally related as lines that pass through the opposite vertices of a cross-polytope, rather rays with an origin in the center of a regular simplex passing through its vertices. It seems related to criteria of simplicity not just a mere protest against an omnipresent orthogonalism. For the polysigned number, equivalent words could be, tergonal for the case with three signs, tetragonal for the case with four signs, etc. He uses the method of parallels and presents a property called the signon.

Golden does not leave it as a coordinate system. It is observed in his web where he mentions the word "vector" a few times, also "unidirectional segments". He highlights the Cancellation law. Also he provides an specific product and promotes the MU or minus unity.

It can be observed in threads starting in 2003, that polysigns, to a good extent, has been developed in the open. To refer to some specific polysigned space, Golden usually writes Pn where n is the number of signs. In case of conflict with some other use, may be used the 𝔾n as well.

In this succinct document will be briefly showed a certain product that can be endowed to polysigned numbers with n > 2.
At various places, modification or change of notation is subject to debate.

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In this document is used the full-width style of @ - + ＊ # for the sign symbols. As a way of proposal, since several symbols can be chosen to continue the stroke's nemonics of Golden. Instead of 兆 one may use 卌, or instead of @ one may use the no-mind symbol.

Some polysigned considerations

If addition is represented under the Golden's metaphor as a zero-like symbol @ then one can write a p4 number as

$$\mathbf{w := -a\ @\ +b\ @\ \ast c\ @\ \# d}$$

or optionally, in quadray's style [1]

$$\mathbf{w := (a,b,c,d)}$$

Multiplication between a magnitude and a p4 number (magnelar multiplication)

$$\mathbf{\mu.(-a\ @\ +b\ @\ \ast c\ @\ \# d) = -\mu a\ @\ +\mu b\ @\ \ast \mu c\ @\ \# \mu d}$$
$$\mathbf{\mu.(a,b,c,d) = (\mu a,\mu b,\mu c,\mu d)}$$

Optional form for the symmetric element of the units in p4

$$\mathbf{\sim\# 1 := -1\ @\ +1\ @\ \ast 1}$$
$$\mathbf{\sim\ast 1 := -1\ @\ +1\ @\ \# 1}$$
$$\mathbf{\sim+1 := -1\ @\ \ast 1\ @\ \# 1}$$
$$\mathbf{\sim-1 := +1\ @\ \ast 1\ @\ \# 1}$$

Magnitude of a certain p4 number

$$\mathbf{/w/^2 := a^2 + b^2 + c^2 + d^2 -(2/3)(ab + ac + ad + bc + bd + cd)}$$

In the above statement the symbol + and - plays the role of increment and decrement

Absolute value of a certain p4 number

$$\mathbf{|w|^2 := \# a^2\ @\ \# b^2\ @\ \# c^2\ @\ \# d^2\ @\ \sim(2/3).(\# ab\ @\ \# ac\ @\ \# ad\ @\ \# bc\ @\ \# bd\ @\ \# cd)}$$

Equation of a centered unit sphere

$$\mathbf{x^2 + y^2 + z^2 + w^2 -(2/3)(xy + xz + xw + yz + yw + zw) = 1}$$

Look [1], [2] and section 5 of [3].

Note that a polysign number may be simplified to its minimal form using the cancellation law, hence at least one variable get eliminated, with the result of being able to easily plot the centered sphere equation.

In Golden’s notation the symbol @ is interchangeable with the bigger sign in the polysigned space in use.

Candidate ways to emphasise the quality of direction

$\overrightarrow{+}\mathbf{a}$ $\hat{+}\mathbf{a}$ ⇷**a**

For operate elements of various polysign spaces more type of arrows may be required (↬ ⇝ ➤ ⤚ ⇢ ⇾)

Case for the pacman product ᗧ in p4

If a relation ⇒ is used (Continens ⇒ Contentum) instead of =, and a simple rule is designed to build a product where # is the neutral sign and each non-neutral sign visits the others non-neutral signs once, to end the cycle in the neutral sign, we may get a product of cyclic behaviour, different than the traditional Golden product for p4, which is also of cyclic nature. It can be interpreted as the minus unity implemented in other way.

(−1)(−1) ⇒ +1 , (+1)(−1) ⇒ ∗1 , (∗1)(−1) ⇒ #1
(+1)(+1) ⇒ ∗1 , (∗1)(+1) ⇒ −1 , (−1)(+1) ⇒ #1
(∗1)(∗1) ⇒ −1 , (−1)(∗1) ⇒ +1 , (+1)(∗1) ⇒ #1

with the center

(−1)(#1) ⇒ −1 , (#1)(−1) ⇒ −1
(+1)(#1) ⇒ +1 , (#1)(+1) ⇒ +1
(∗1)(#1) ⇒ ∗1 , (#1)(∗1) ⇒ ∗1
(#1)(#1) ⇒ #1

ᗧ	−	+	∗	#
−	+	#	+	−
+	∗	∗	#	+
∗	#	−	−	∗
#	−	+	∗	#

(~#1)(~#1) ⇒ #1

Several properties of the p4 pacman product under the ⇒ relation

not commutative
(−3)(+2) ⇒ #6
(+2)(−3) ⇒ ∗6

not associative
(−2)((+3)(∗5)) ⇒ (−2)(#15) ⇒ −30
((−2)(+3))(∗5) ⇒ (#6)(∗5) ⇒ ∗30

not flexible
(−3)((∗2)(−3)) ⇒ (−3)(#6) ⇒ −18
((−3)(∗2))(−3) ⇒ (+6)(−3) ⇒ ∗18

not left alternative
(−2)((−2)(+5)) ⇒ (−2)(#10) ⇒ −20
((−2)(−2))(+5) ⇒ (+4)(+5) ⇒ ∗20

not right alternative
((+7)(−3))(−3) ⇒ (∗21)(−3) ⇒ #63
(+7)((−3)(−3)) ⇒ (+7)(+9) ⇒ ∗63

not power-associative
(−2)((−2)(−2)) ⇒ (−2)(+4) ⇒ #8
((−2)(−2))(−2) ⇒ (+4)(−2) ⇒ ∗8
notation w^n would be ambiguous for **n>2**

if the usual equality was valid to use (not the case) then,
with such equality would be observed that the cancellative property does not apply
(+2)(−5) = (+2)(+5) -> (−5) = (+5) ?

does not satisfy the zero-product property
(−1 @ #1)(#1 @ ∗1) ⇒ −1 @ +1 @ #1 @ ∗1 ⇒ 0

examples of an element that has left and right reciprocal
(∗0,2)(−5) ⇒ #1
(−5)(+0,2) ⇒ #1

The table of the product is not a latin square (some sort of quasi-latin square),
and after repeatedly multiplying on the left may yield a neutral-signed term

(−1)(−a @ +b @ ∗c @ #d) ⇒ +a @ #b @ +c @ −d
(+1)(+a @ #b @ +c @ −d) ⇒ ∗a @ +b @ ∗c @ ∗d
(∗1)(∗a @ +b @ ∗c @ ∗d) ⇒ −a @ −b @ −c @ −d
(∗1)(−a @ −b @ −c @ −d) ⇒ #a @ #b @ #c @ #d

The price of this new product is, so far, pretty expensive.

Conjugates of a p4 number under the pacman product

w := −a @ +b @ ∗c @ #d   a p4 number
w' := +a @ ∗b @ −c @ #d   first pacman conjugate
w" := ∗a @ −b @ +c @ #d   second pacman conjugate
w° := #a @ #b @ #c @ #d   neutralized pacman conjugate

Components of a p4 number in function of its pacman conjugates

#a := (1/4).((w)(+1) @ (w')(∗1) @ (w")(−1) @ (w°)(#1))
#b := (1/4).((w)(∗1) @ (w')(−1) @ (w")(+1) @ (w°)(#1))
#c := (1/4).((w)(−1) @ (w')(+1) @ (w")(∗1) @ (w°)(#1))
#d := (1/4).((w)(#1) @ (w')(#1) @ (w")(#1) @ (w°)(#1))

The following property relates a number and its conjugates to the absolute value
(1/3).(ww' @ w'w" @ w"w) ⇒ #a^2 @ #b^2 @ #c^2 @ #d^2 @ ~(2/3).(#ab @ #ac @ #ad @ #bc @ #bd @ #cd)

With the neutralized conjugate included
(1/4).(ww' @ w'w" @ w"w @ w°w°) ⇒ #a^2 @ #b^2 @ #c^2 @ #d^2

Regarding four-signed dot-like products derived from the pacman product

t := −t1 @ +t2 @ ∗t3 @ #t4
w := −w1 @ +w2 @ ∗w3 @ #w4

A weak dot-like product

t•w := (tw' @ t'w" @ t"w @ t°w°)/4 ⇒ #t1w1 @ #t2w2 @ #t3w3 @ #t4w4
w•t := (wt' @ w't" @ w"t @ w°t°)/4 ⇒ #t1w1 @ #t2w2 @ #t3w3 @ #t4w4

•	−1	+1	∗1	#1
−1	#1	0	0	0
+1	0	#1	0	0
∗1	0	0	#1	0
#1	0	0	0	#1

•^	−1	+1	∗1	#1
−1	1	0	0	0
+1	0	1	0	0
∗1	0	0	1	0
#1	0	0	0	1

The table of the left is extrapolated from t•w, while the table of the right is a manufactured to resemble the usual dot product, yielding a "scalar". Tables just for illustrative purposes.

A strong dot-like product

t•w := (tw' @ t'w" @ t"w)/3
w•t := (wt' @ w't" @ w"t)/3
⇒ #t1w1 @ #t2w2 @ #t3w3 @ #t4w4 @ 2/3.~(#t1w2 @ #t1w3 @ #t1w4 @ #t2w1 @ #t2w3 @ #t2w4 @ #t3w1 @ #t3w2 @ #t3w4 @ #t4w1 @ #t4w2 @ #t4w3)

•	−1	+1	∗1	#1
−	#1	~#2/3	~#2/3	~#2/3
+	~#2/3	#1	~#2/3	~#2/3
∗	~#2/3	~#2/3	#1	~#2/3
#	~#2/3	~#2/3	~#2/3	#1

•^	−1	+1	∗1	#1
−	+1	-2/3	-2/3	-2/3
+	-2/3	+1	-2/3	-2/3
∗	-2/3	-2/3	+1	-2/3
#	-2/3	-2/3	-2/3	+1

The table of the left is extrapolated from t•w, while the table of the right is a manufactured to resemble the usual dot product, yielding a "scalar" Tables just for illustrative purposes.

This last dot-like product in particular satisfies $|w|^2 := w•w$
Note that is not just the sum of squares.

Also note that a statement $|w|^2 := w•w$ is different compared to |a||b| = |ab|. On one side there are some limitative guidelines, on the other side, it is not excluded the existence of some sort of analog of |a||b| = |ab|, however, of distinct nature.

To illustrate the cyclical nature of the pacman product look at the following arbitrary products, compared to some anticommuting product.
[u,v,w]   := uv @ vw @ wu
[t,u,v]   := (tu)v @ (uv)t @ (vt)u
[t,u,v]   := (tu')v" @ (uv')t" @ (vt')u"
[t,u,v,w] := ((tu)v)w @ ((uv)w)t @ ((vw)t)u @ ((wt)u)v

Case for the pacman product ᗧ in p5

(−1)(−1) ⇒ +1 , (+1)(−1) ⇒ ∗1 , (∗1)(−1) ⇒ #1 , (#1)(−1) ⇒ 生1
(+1)(+1) ⇒ ∗1 , (∗1)(+1) ⇒ #1 , (#1)(+1) ⇒ −1 , (−1)(+2) ⇒ 生1
(∗1)(∗1) ⇒ #1 , (#1)(∗1) ⇒ −1 , (−1)(∗1) ⇒ +1 , (+1)(∗3) ⇒ 生1
(#1)(#1) ⇒ −1 , (−1)(#1) ⇒ +1 , (+1)(#1) ⇒ ∗1 , (∗1)(#1) ⇒ 生1

with the center

(−1)(生1) ⇒ −1 , (生1)(−1) ⇒ −1
(+1)(生1) ⇒ +1 , (生1)(+1) ⇒ +1
(∗1)(生1) ⇒ ∗1 , (生1)(∗1) ⇒ ∗1
(#1)(生1) ⇒ #1 , (生1)(#1) ⇒ #1
(生1)(生1) ⇒ 生1

ᗧ	－	＋	＊	＃	生
－	＋	生	＋	＋	－
＋	＊	＊	生	＊	＋
＊	＃	＃	＃	生	＊
＃	生	－	－	－	＃
生	－	＋	＊	＃	生

w   := −a @ +b @ ∗c @ #d @ 生e
w' := +a @ ∗b @ #d @ −d @ 生e
w" := ∗a @ #b @ −c @ +d @ 生e
w‴ := #a @ −b @ +c @ ∗d @ 生e
w° := 生a @ 生b @ 生c @ 生d @ 生e

(1/4).(ww' @ w'w" @ w"w‴ @ w‴w) ⇒ 生a^2 @ 生b^2 @ 生c^2 @ 生d^2 @ 生e^2 @
(2/4).~(生ab @ 生ac @ 生ad @ 生ae @ 生bc @ 生bd @ 生be @ 生cd @ 生ce @ 生de)

(1/5).(ww' @ w'w" @ w"w‴ @ w‴w @ w°w°) ⇒ 生a^2 @ 生b^2 @ 生c^2 @ 生d^2 @ 生e^2

It is observed that the orbit of each unit under this product has some resemblance compared to the additive orbit of the Golden product for p5, however, the pacman product in p5 is not a latin square, while the last satisfy the latin square arrangement.

Case for the pacman product ᗧ in p6

(−1)(−1) ⇒ +1 , (+1)(−1) ⇒ ∗1 , (∗1)(−1) ⇒ #1 , (#1)(−1) ⇒ 生1 , (生)(−1) ⇒ 兆1
(+1)(+1) ⇒ ∗1 , (∗1)(+1) ⇒ #1 , (#1)(+1) ⇒ 生1 , (生1)(+1) ⇒ −1 , (−1)(+1) ⇒ 兆1
(∗1)(∗1) ⇒ #1 , (#1)(∗1) ⇒ 生1 , (生1)(∗1) ⇒ −1 , (−1)(∗1) ⇒ +1 , (+1)(∗1) ⇒ 兆1
(#1)(#1) ⇒ 生1 , (生1)(#1) ⇒ −1 , (−1)(#1) ⇒ +1 , (+1)(#1) ⇒ ∗1 , (∗1)(#1) ⇒ 兆1
(生1)(生1) ⇒ −1 , (−1)(生1) ⇒ +1 , (+1)(生1) ⇒ ∗1 , (∗1)(生1) ⇒ #1 , (#1)(生1) ⇒ 兆1

with the center

(−1)(兆1) ⇒ −1 , (兆1)(−1) ⇒ −1
(+1)(兆1) ⇒ +1 , (兆1)(+1) ⇒ +1
(∗1)(兆1) ⇒ ∗1 , (兆1)(∗1) ⇒ ∗1
(#1)(兆1) ⇒ #1 , (兆1)(#1) ⇒ #1
(生1)(兆1) ⇒ 生1, (兆1)(生1) ⇒ 生1
(兆1)(兆1) ⇒ 兆1

ᗧ	－	＋	＊	＃	生	兆
－	＋	兆	＋	＋	＋	－
＋	＊	＊	兆	＊	＊	＋
＊	＃	＃	＃	兆	＃	＊
＃	生	生	生	生	兆	＃
生	兆	－	－	－	－	生
兆	－	＋	＊	＃	生	兆

w := －a @ ＋b @ ＊c @ ＃d @ 生e @ 兆f
w′ := ＋a @ ＊b @ ＃c @ 生d @ －e @ 兆f
w″ := ＊a @ ＃b @ 生c @ －d @ ＋e @ 兆f
W‴ := ＃a @ 生b @ －c @ ＋d @ ＊e @ 兆f
w⁗ := 生a @ －b @ ＋c @ ＊d @ ＃e @ 兆f
w° := 兆a @ 兆b @ 兆c @ 兆d @ 兆e @ 兆f

(1/5).(ww′ @ w′w″ @ w″w‴ @ w‴w⁗ @ w⁗w) ⇒ 兆a^2 @ 兆b^2 @ 兆c^2 @ 兆d^2 @ 兆e^2 @ 兆f^2
@ (2/5).~(兆ab @ 兆ac @ 兆ad @ 兆ae @ 兆af @兆bc @ 兆bd @
兆be @ 兆bf @ 兆cd @ 兆ce @ 兆cf @ 兆de @ 兆df @ 兆ef)

(1/6).(ww′ @ w′w″ @ w″w‴ @ w‴w⁗ @ w⁗w @ w°w°) ⇒ 兆a^2 @ 兆b^2 @ 兆c^2 @ 兆d^2 @ 兆e^2 @ 兆f^2

Case for the pacman product ᗧ in p3

(－1)(－1) ⇒ ＋1 , (＋1)(－1) ⇒ ＊1
(＋1)(＋1) ⇒ －1 , (－1)(＋1) ⇒ ＊1

with center

(－1)(＊1) ⇒ －1 , (＊1)(－1) ⇒ －1
(＋1)(＊1) ⇒ ＋1 , (＊1)(＋1) ⇒ ＋1
(＊1)(＊1) ⇒ ＊1

It is observed that this product coincide with the Golden product for p3 and represents an special case

ᗧ	－	＋	＊
－	＋	＊	－
＋	＊	－	＋
＊	－	＋	＊

w := －a @ ＋b @ ＊c
w' := ＋a @ －b @ ＊c
w° := ＊a @ ＊b @ ＊c

(1/2).(ww' @ w'w) ⇒ ＊a^2 @ ＊b^2 @ ＊c^2 @ (2/2).~(＊ab @ ＊ac @ ＊bc)
(1/2).(ww' @ w'w) ⇒ ww'
(1/3).(ww' @ w'w @ w°w°) ⇒ ＊a^2 @ ＊b^2 @ ＊c^2
|w|^2 := w•w

Miscellaneous regarding to arbitrary products

With **|(1,1,1,0| = 1**, is it possible to considerate a product's rule satisfiying **(-1)(+1) ⇒ ~#1** ?
With **|((sqrt(3/4) , sqrt(3/4) , 0 , 0)| = 1**, is it possible consider a product' rule satisfying **(-1)(+1) ⇒ sqrt(3/4).(*1** @ **#1)** ?
Is it necessary to keep the property **(~#1)(~#1) ⇒ #1** ?
What considerations of symmetry must be taken into account while observing the tables of arbitrary products ?

In case of designing a product without trivial zero-divisors, identity element, center, nilpotent or idempotent, how to check the existence of non-trivial artifacts ?

Several considerations regarding the dot and cross products can be tracked to the quaternion structure, and examining how the vectorial and scalar "parts" interact. For general enquiry [5]. Also look at [6] and [7].

The author has not found a relation to link the pacman product with a possible cross-like product. A candidate for a cross-like product is presented.

×	−	+	∗	#
−	0	#	+	∗
+	∗	0	#	−
∗	#	−	0	+
#	+	∗	−	0

¤	−	+	∗	#
−	−	#	+	∗
+	∗	+	#	−
∗	#	−	∗	+
#	+	∗	−	#

Resources

1. Kirby Urner (October 7, 1997 / March 25, 2000), http://www.grunch.net/synergetics/quadintro.html
2. Hagen von Eitzen (February 23, 2009), "Understanding Polysign Numbers the Standard Way" http://www.von-eitzen.de/math/PolysignNumbers.pdf
3. Spoonfed, Golden (May 28, 2006 / Jun 25, 2006) thread "Matrix form for polysign product" https://groups.google.com/g/sci.math/c/WwVcOf1gs3M/m/dPAszuW3VVgJ}
5. John Vince (2011), "Quaternions for computer graphics"
6. Simon Altmann (December, 1989), "Hamilton, Rodrigues and the Quaternion Scandal"
7. Whittaker (1944 and 1945), "The Sequence of Ideas in the Discovery of Quaternions"
8. Florian Cajori (1928 and 1929), "A history of mathematical notations" Volume I and II
9. Timothy Golden, "Golden's Homepage" http://www.bandtechnology.com/

バッタ たなか signing off 。

July + @ + -

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